Free surface flows with ALE in *Code_Saturne*

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Overview

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Free surface boundary condition

Kinematic Boundary condition

$$\dot{m}_{FS} = 0$$

Dynamic Boundary condition

$$\mathbf{n} \cdot \boldsymbol{\Gamma} \cdot \mathbf{n}^T = \frac{1}{We}\left(\frac{1}{R_1} + \frac{1}{R_2}\right)$$

$$\mathbf{t_1} \cdot \boldsymbol{\Gamma} \cdot \mathbf{n}^T = 0$$

$$\mathbf{t_2} \cdot \boldsymbol{\Gamma} \cdot \mathbf{n}^T = 0$$

with $\boldsymbol{\Gamma} = -P\mathbf{I} + \dfrac{1}{Re}\left(\nabla\mathbf{u} + (\nabla\mathbf{u})^T\right)$

Expansion of the tangential stress condition

Example on $\mathbf{t_1}$

$$\begin{bmatrix} t_1\left(-P+\frac{2}{Re}\frac{\partial u}{\partial x}\right)+t_2\frac{1}{Re}\left(\frac{\partial u}{\partial y}+\frac{\partial v}{\partial x}\right)+t_3\frac{1}{Re}\left(\frac{\partial u}{\partial z}+\frac{\partial w}{\partial x}\right) \\ t_1\frac{1}{Re}\left(\frac{\partial u}{\partial y}+\frac{\partial v}{\partial x}\right)+t_2\left(-P+\frac{2}{Re}\frac{\partial v}{\partial y}\right)+t_3\frac{1}{Re}\left(\frac{\partial v}{\partial z}+\frac{\partial w}{\partial y}\right) \\ t_1\frac{1}{Re}\left(\frac{\partial u}{\partial z}+\frac{\partial w}{\partial x}\right)+t_2\frac{1}{Re}\left(\frac{\partial v}{\partial z}+\frac{\partial w}{\partial y}\right)+t_3\left(-P+\frac{2}{Re}\frac{\partial w}{\partial z}\right) \end{bmatrix}\cdot\begin{bmatrix} n_1 \\ n_2 \\ n_3 \end{bmatrix}=0$$

$$\begin{aligned} t_1 n_1\left(-P+\frac{2}{Re}\frac{\partial u}{\partial x}\right)+t_2 n_2\frac{1}{Re}\left(\frac{\partial u}{\partial y}+\frac{\partial v}{\partial x}\right)+t_3 n_1\frac{1}{Re}\left(\frac{\partial u}{\partial z}+\frac{\partial w}{\partial x}\right) &\quad + \\ t_1 n_2\frac{1}{Re}\left(\frac{\partial u}{\partial y}+\frac{\partial v}{\partial x}\right)+t_2 n_2\left(-P+\frac{2}{Re}\frac{\partial v}{\partial y}\right)+t_3 n_2\frac{1}{Re}\left(\frac{\partial v}{\partial z}+\frac{\partial w}{\partial y}\right) &\quad + \\ t_1 n_3\frac{1}{Re}\left(\frac{\partial u}{\partial z}+\frac{\partial w}{\partial x}\right)+t_2 n_3\frac{1}{Re}\left(\frac{\partial v}{\partial z}+\frac{\partial w}{\partial y}\right)+t_3 n_3\left(-P+\frac{2}{Re}\frac{\partial w}{\partial z}\right) &\quad = \quad 0 \end{aligned}$$

Special case (1)

Bi-dimensional case in $x - z$ plane

$$\mathbf{n} = \begin{bmatrix} -\eta_x/\mathcal{N} \\ 1/\mathcal{N} \end{bmatrix} \quad \mathbf{t_1} = \begin{bmatrix} 1/\mathcal{N} \\ \eta_x/\mathcal{N} \end{bmatrix}$$

with $\mathcal{N} = \sqrt{1+\eta_x^2}$

$$\frac{1}{Re}\,\frac{1}{\mathcal{N}}\left[-2\eta_x\left(\frac{\partial u}{\partial x} - \frac{\partial w}{\partial z}\right) + (1-\eta_x^2)\left(\frac{\partial u}{\partial z} + \frac{\partial w}{\partial x}\right)\right] = 0$$

$$p = \frac{\eta}{Fr^2} + \frac{1}{Re}\,\frac{2}{\mathcal{N}}\left[\eta_x^2\frac{\partial u}{\partial x} + \frac{\partial w}{\partial z} - \eta_x\left(\frac{\partial u}{\partial x} + \frac{\partial w}{\partial x}\right)\right] + \frac{\eta_{xx}}{\mathcal{N}^3}\,\frac{\sigma}{We}$$

N.B.: This BC is applied at the actual free surface (i.e. $z = h + \eta$)

Special case (2)

Orthogonal mesh with $g_z \neq 0$ and flow in x

$$\mathbf{n} = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \quad \mathbf{t_1} = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} \Rightarrow \frac{1}{Re}\left(\frac{\partial u}{\partial z} + \frac{\partial w}{\partial x}\right) = 0$$

$$\frac{1}{Re}\left(\frac{\partial v}{\partial z} + \frac{\partial w}{\partial y}\right) = 0 \Rightarrow \frac{\partial u_i}{\partial n} = 0 \text{ with } \nu \to 0$$

$$p = \frac{\eta}{Fr^2} + \frac{2}{Re}\frac{\partial w}{\partial z} \Rightarrow p = \frac{\eta}{Fr^2} \text{ with } \nu \to 0$$

N.B.: This BC is applied at the undisturbed free surface (i.e. $z = h \Rightarrow$ linearisation)

Rigid lid approximation

$$\frac{\partial u_1}{\partial x_2} = \frac{\partial u_3}{\partial x_2} = \frac{\partial p}{\partial x_2} = u_2 = 0$$

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Overview

Method to compute the mesh velocity $\underline{w}$

Solve a Poisson equation

$$\begin{cases} \underline{\mathrm{div}}\left(\underline{\underline{\lambda}}\,\underline{\underline{\mathrm{grad}}}\;\underline{w}\right) & = & 0 \\ \underline{w}\big|_{\Gamma(t)} & = & \text{imposed velocity} \\ \left.\dfrac{\partial\left(\underline{w} - \underline{w}.\underline{n}\,\underline{n}\right)}{\partial n}\right|_{\partial\Omega\backslash\Gamma(t)} & = & \underline{0} \\ \underline{w}.\underline{n}\big|_{\partial\Omega\backslash\Gamma(t)} & = & 0 \end{cases}$$

ALE algorithm

for n to $n+1$ **do**

Set $\underline{w}$-Boundary Conditions on $\Gamma(t^{n+1})$

Compute $\underline{w}_I^{n+1}$ by solving the Poisson equation

Reconstruct mesh velocity at **nodes** $\underline{w}_N^{n+1}$

Update the mesh from Ω^n to Ω^{n+1} with $\underline{w}_N^{n+1}$

end for

Method to compute the mesh velocity $\underline{w}$

Solve a Poisson equation

$$
\left\{
\begin{array}{rcl}
\underline{\mathrm{div}}\left(\underline{\underline{\lambda}}\,\underline{\mathrm{grad}}\ \underline{w}\right) & = & 0 \\
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\end{array}
\right.
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end for

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New ALE algorithm in *Code_Saturne*

Algorithm 1 Implementation of the ALE module with free surfacein *Code_Saturne* V_3.0

for n to $n+1$ **do**
 Compute total mass flux $\rho\left(\underline{u}-\underline{w}\right)_f^n.\underline{S}^n$
 Compute the predicted velocity $\underline{\widetilde{u}}_I^{n+1}$ on Ω^n
 Compute the pressure corrected velocity $\underline{u}_I^{n+1}$ on Ω^n
 Compute mass flux $\underline{u}_f^{n+1}.\underline{S}^n$
 Compute $\underline{w}_I^{n+1}$ with free-surface BCs $\underline{u}_f^{n+1}.\underline{S}^n$
 Reconstruction mesh velocities at nodes $\underline{w}_N^{n+1}$
 Update the mesh from Ω^n to Ω^{n+1}
end for

Algorithm 2 Implementation of the ALE module with free surfacein *Code_Saturne* V_3.1

for n to $n+1$ **do**
 Solve the NS and in the Algorithm 1
 Reset mesh to initial time 0
 Evaluate ALE BC in order to move the mesh from $0 \rightarrow n+1$
 Solve the ALE on mesh at time 0 with BC at $n+1$
 Reconstruction mesh velocities at nodes $\underline{w}_N^{n+1}$
 Update the mesh from 0 to Ω^{n+1}
end for

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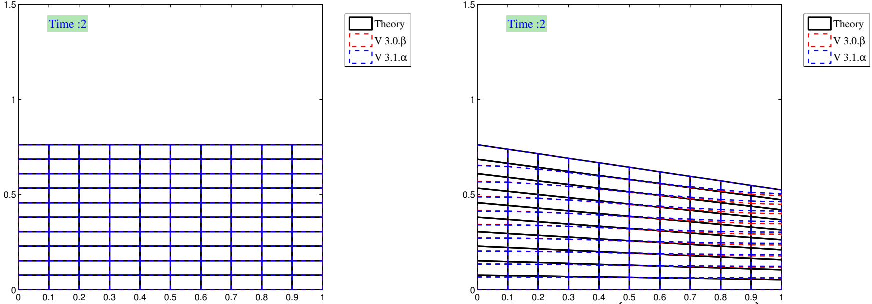
Verification with imposed mesh movement (linear piston)



(a) $\eta_{FS} = a \cdot sin(\omega t)$ (b) $\eta_{FS} = a\left(\frac{x - x_{min}}{x_{max}} + 1\right)\sin(\omega t)$

Figure: Free surface movement externally imposed

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Verification with imposed mesh movement (linear piston)

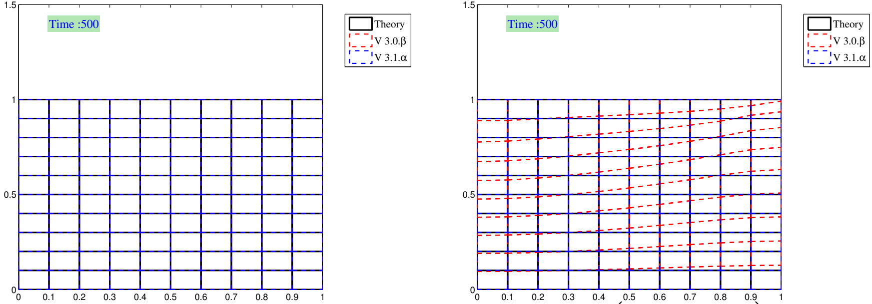

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Figure: Free surface movement externally imposed

Overview

Definition

Inlet definition

$$\eta = \frac{A}{\cosh^2(kx - \omega t)} \qquad c = \frac{\omega}{k} \qquad d = 1\,m$$

$$u = \frac{c\eta}{d+\eta} \qquad \omega = \frac{2\pi}{T} \qquad A = 0.2\,m$$

$$w = \frac{2kdu\tanh(kx - \omega t)}{d+\eta} z \qquad k = \frac{2\pi}{\lambda} \qquad g = 9.81\,m/s^2$$

Convective outlet

$$\frac{\partial \Phi}{\partial t} + U\frac{\partial \Phi}{\partial n} = 0 \Rightarrow \frac{\Phi_F^{n+1} - \Phi_F^n}{\Delta t} + U\frac{\Phi_F^{n+1} - \Phi_I^{n+1}}{\left\|\overline{IF}\right\|} = 0$$

$$\Phi_F^{n+1} = \frac{1}{1+CFL}\Phi_F^n + \frac{CFL}{1+CFL}\Phi_I^{n+1} \text{ with } CFL = \frac{U\Delta t}{\left\|\overline{IF}\right\|}$$

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Code_Saturne V.2.3 vs V.2.0

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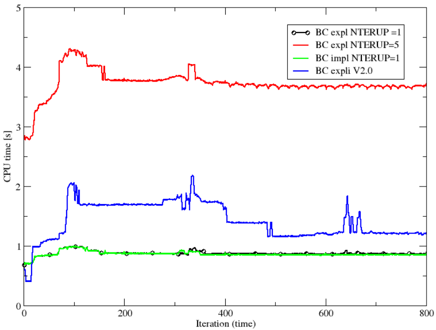
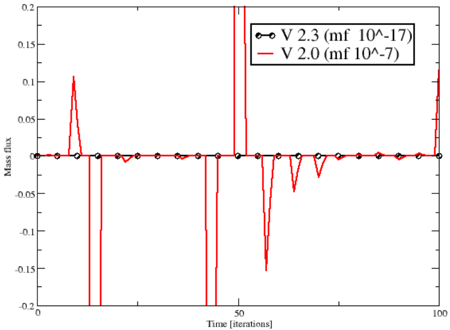
Code_Saturne V.2.3 vs V.2.0





Overview

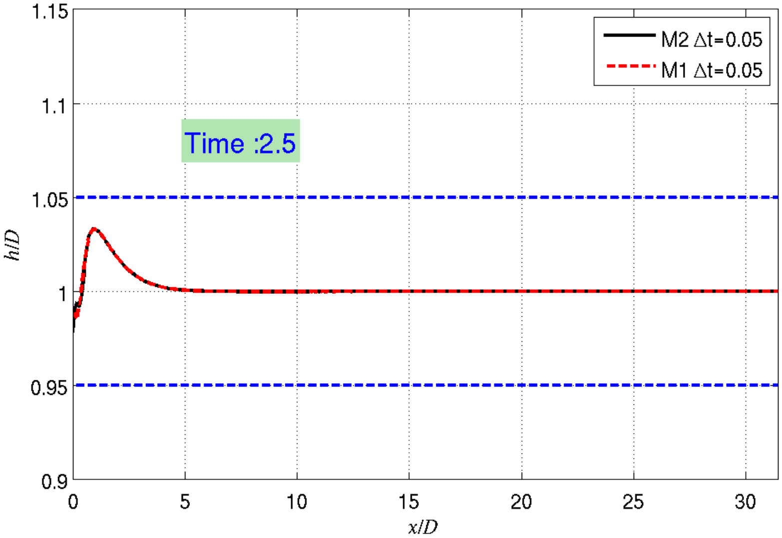
Test case definition

Inlet definition

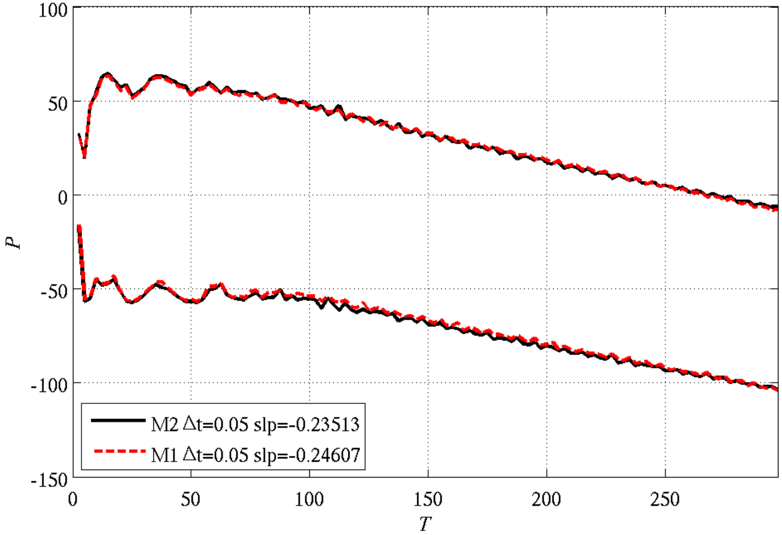
$$u = A\frac{gk}{\omega}\cos\left(kx-\omega t\right)\frac{\cosh\left(kz\right)}{\sinh\left(kd\right)}$$
$$w = A\frac{gk}{\omega}\sin\left(kx-\omega t\right)\frac{\sinh\left(kz\right)}{\sinh\left(kd\right)}$$
$$\omega^2 = gk\tanh\left(kd\right)$$

	Dimensions $x \times z$	Mesh resolution
M1	$10\pi \times 1$	500×20
M2	$10\pi \times 1$	1000×40
M3	$50\pi \times 1$	2500×20
M4	660×45	Several

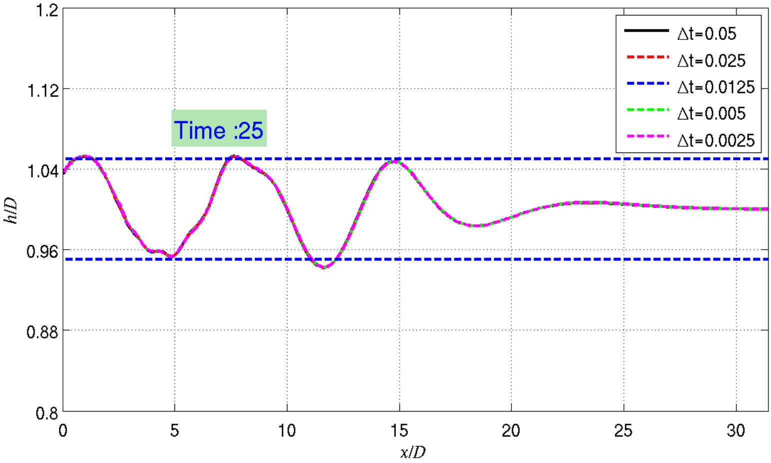
M1 vs. M2: free surface elevation



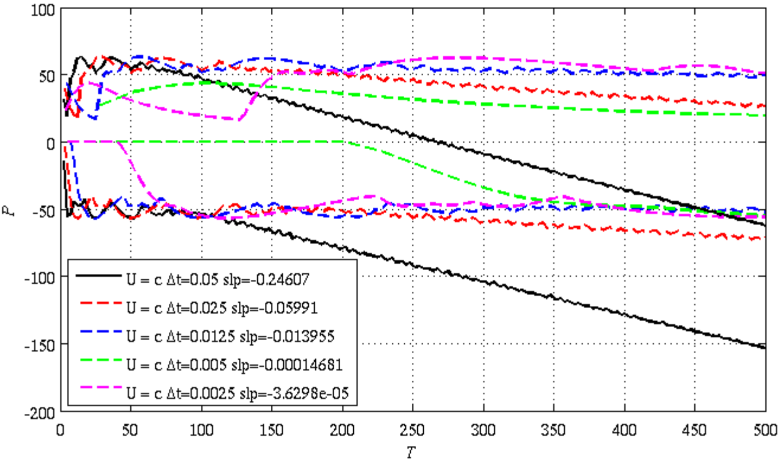
M1 vs. M2: pressure history



Time step variation: free surface elevation



Time step variation: pressure history



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Corrections definitions

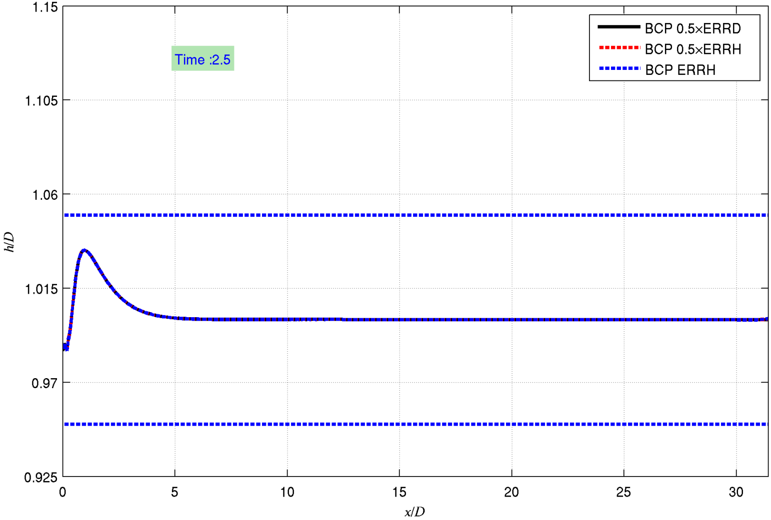
	Inlet	Outlet
BCP	$p = p - \frac{c\Delta t}{L_x}\rho g \Delta$	$p = p + \frac{c\Delta t}{L_x}\rho g \Delta$
QDM	$\frac{dp}{dx} = -\rho g \frac{\Delta}{l}$	

	Def	Ref l
ΔH	$h_{th}(t) - h(t)$	k
ΔD	$d - d_{avg}$	d

Running average definition

$$\langle d \rangle^n = \frac{\Delta t}{aT} h^n + \left(1 - \frac{\Delta t}{aT}\right) \langle d \rangle^{n-1}$$

BCP correction: free surface



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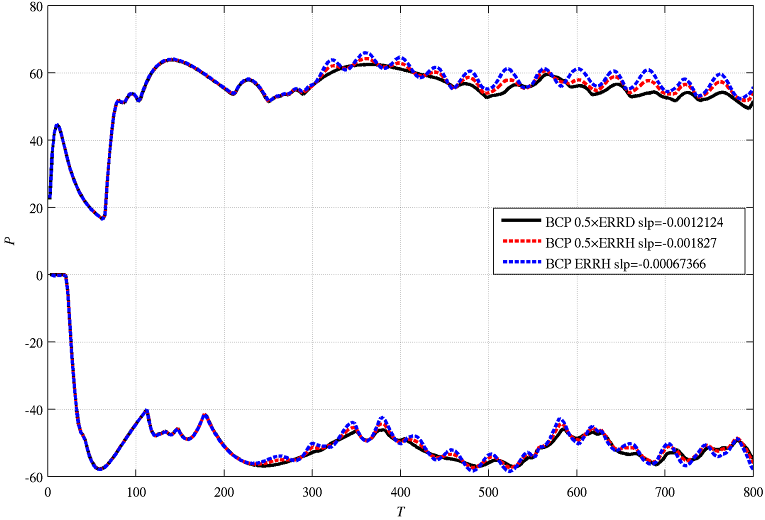
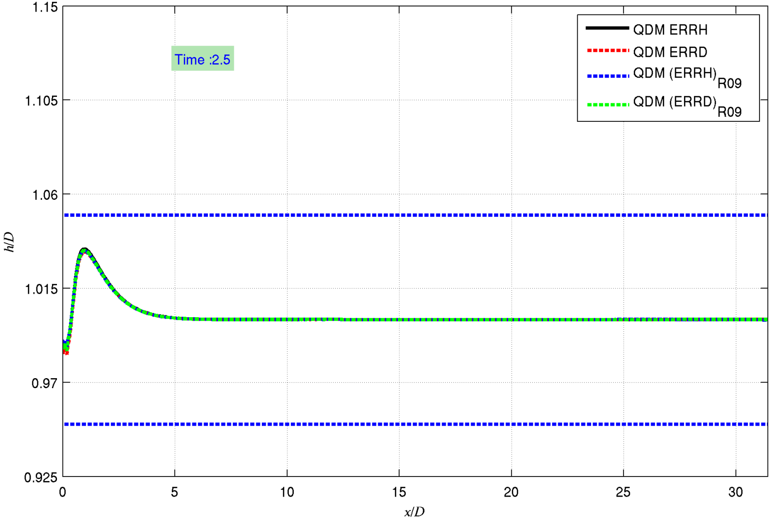
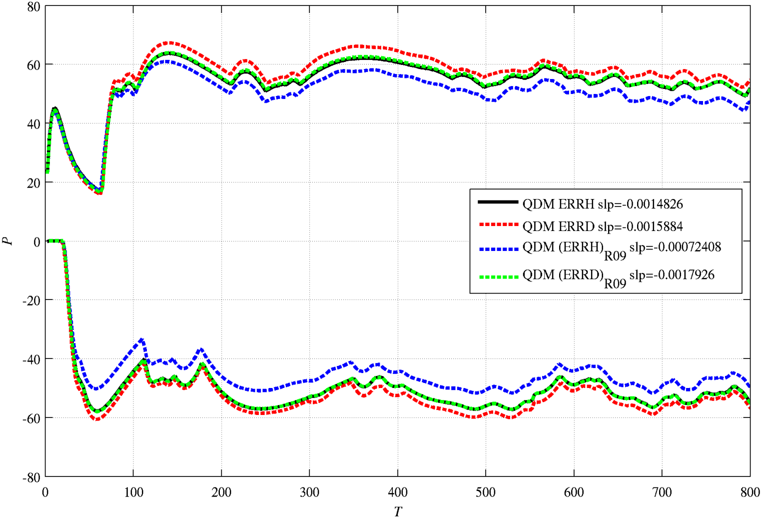
BCP: pressure history



QDM correction: free surface



QDM: pressure history



Overview

Conclusions & Future work

A V&V procedure of the ALE using free surface flows has been conducted

Findings

- Verification of the ALE using inviscid wave propagation (solilatary and regular waves)
- Introduction in *Code_Saturne* of a convective outlet
  - Problem of mass conservation with very long time simulations
- Viscus calculation of open channel at several Reynolds number

Future work

- Extension of the FS BC with the inclusion of viscous terms
- Extension of the open channel to responding free-surface

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Acknowledgements

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